

Multi-level preconditioning in electrodiffusion and poroelasticity

The first part is about the immersed boundary method with advection-electrodiffusion, a mathematical and computational framework of fluid-solute-structure interaction. The redistribution of ionic concentration according to a possible movement of a subcellular membrane is realized replacing Dirichlet / Neumann interface conditions by regularized chemical potentials along the membrane. Stiff gradients around the membrane are well-captured with an implicit second-order Godunov upwind scheme. With local mesh refinement, the GMRES was incorporated as the main solver with the associated non-compact 9-points stencil on 2D. A fast adaptive composite-grid (FAC) method is applied for a preconditioner with a geometric bottom solver, *i.e.*, with the compact 5-point stencil. This scheme is a foundation of the ongoing progress in simulating synaptic structural plasticity and cell migration.

In the second part, we consider the Biot model with block preconditioners and generalized eigenvalue problems for scalability and robustness to parameters. A discontinuous Galerkin discretization is employed with the displacement and Darcy flow flux discretized as piecewise continuous in P_1 elements and the fluid pressure as piecewise constant in the P_0 element with a stabilizing term. Parallel algorithms are designed to solve the resulting linear system. Specifically, the GMRES method is employed as the outer iteration algorithm and block-triangular preconditioners are designed to accelerate the convergence. In the preconditioners, the approximate Schur complement is solved by the two-level additive overlapping Schwartz method where coarse grids are constructed by generalized eigenvalue problems in the overlaps (GenEO). Extensive numerical experiments are conducted to show the scalability and parametric robustness of the resulting parallel algorithms. The scheme is a foundation of a recently funded DoD lung project.