ALMOST SURE LOCAL WELL-POSEDNESS OF THE NONLINEAR SCHRÖDINGER EQUATION USING DIRECTIONAL ESTIMATES.

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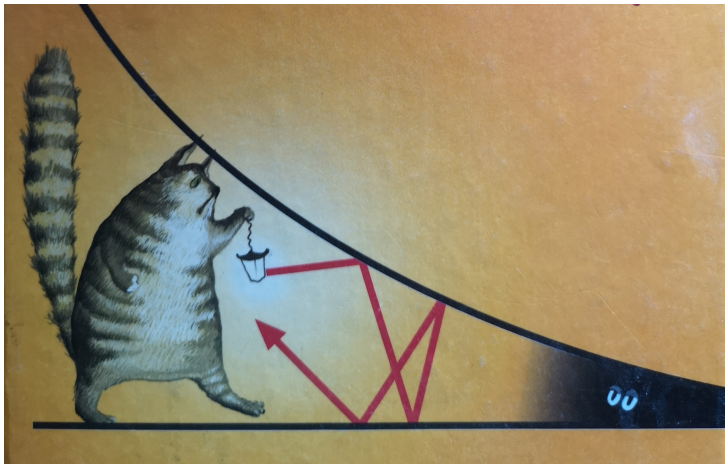

The nonlinear Schrödinger equation (NLS) on $\mathbb{R}^d$ is a prototypical dispersive equation, i.e. it is characterized by different frequencies travelling at different velocities and by the lack of a smoothing effect over time.

Furthermore, NLS is a prototypical infinite-dimensional Hamiltonian system. Constructing an invariant measure for the NLS flow is a natural, albeit very difficult problem. It requires showing local well-posedness in low regularity spaces, in an appropriate probabilistic setting.

Deterministic local well-posedness for the NLS is well-understood: it holds only for initial data with regularity above a certain energy-critical threshold.

We show how directional behavior combined with multilinear tree expansions for the solutions provide the framework to deal with randomized initial data in any positive regularity for the cubic power nonlinearity in dimension 3. This approach improves our understanding of the structure of the solutions and sheds light on NLS in dimensions $d \geq 3$ and potentially with other power nonlinearities.

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