Dissipation Vs. Quadratic Nonlinearity: from Energy Bound to High-Order Regularizing Effect

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We consider a general class of evolutionary PDEs involving dissipation (of possibly fractional order), which competes with quadratic nonlinearities on the regularity of the overall equation. This includes the prototype mod- els of Burgers equation, Navier-Stokes equations, the surface quasi-geostrophic equations and the Keller-Segel model for chemotaxis. Here we establish a Petrowsky type parabolic estimate of such equations which entail a precise time decay of higher-order Sobolev norms for this class of equations. To this end, we introduce as a main new tool, an infinite order energy functional"

$$E(t) := \sum_{n=0}^{\infty} \alpha_n t^n \|(-\Delta)u(*, t)\|_{L^2},$$

which captures the regularizing effect of all higher order derivatives of $u(*, t)$, by proving, for a carefully chosen problem-dependent choice of weights $\{\alpha_n\}$, that $E(t)$ is non-increasing in time. This is a joint work with Eitan Tadmor at University of Maryland, College Park.