Markov Chain Tree Theorem and Related Problems

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This talk is mainly about a remarkable theorem in the theory of finite Markov Chains (MCs) - MC Tree theorem (MCTT), though in the beginning I will mention briefly two of my other results: 1) a Decomposition-Separation Theorem, describing the behaviour of a family of finite nonhomogeneous Markov chains, defined by the sequence of stochastic matrices (P_n), without any assumptions on this sequence; 2) a rather strange looking number $888,888$ equal to the number of pairs of independent events when a fair coin is flipped and a fair die is tossed, and its relationship to a concept of independence.

MCTT is equivalent to a theorem initially discovered by Gustav Kirchhoff, a famous physicist of XIX century, expressing the limiting distribution π for an ergodic MC in terms of directed spanning trees. This theorem provides two "bridges": one between MCs and Graph Theory, and the other between Mathematics and Electrical Engineering. The key role in this theorem is played by vector $q = (q(y), y \in S)$, where y is a point in a state space S, and $q(y)$ is calculated by summation over all spanning trees directed to y of the products of corresponding entries of a stochastic matrix. The applications of MCTT are limited by the fact that the number of trees directed to a point is growing exponentially. In paper (Sonin, 1999) it was noted and proved that there is a polynomial algorithm to calculate $q(y)$ having a simple probabilistic interpretation. The proof was complicated and used some subtle facts from the graph theory. This algorithm and corresponding theorem were generalized into the case of so called idempotent (tropical) calculus in paper (Gursoy et al., 2015). A new proof of this theorem without using any results from the graph theory will be outlined.

References

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