Weakly homogeneous variational inequalities

by

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Abstract: Given a closed convex cone C in a finite dimensional real Hilbert space H, a weakly homogeneous map $f : C \to H$ is a sum of two continuous maps h and g, where h is positively homogeneous of degree γ (> 0) on C and $g(x) = o(||x||^\gamma)$ as $||x|| \to \infty$ in C. The map h, denoted by f^∞, will be called the 'leading term or the recession part' of f. Examples include polynomial maps over R^n and the Riccati map $f(X) := XAX + BX + XB^*$ over the cone of (real/complex) Hermitian positive semidefinite matrices.
Given a weakly homogeneous map f, a nonempty closed convex subset K of C, and a $q \in H$, we consider the variational inequality problem, VI(f, K, q), of finding an $x^* \in K$ such that $\langle f(x^*) + q, x - x^* \rangle \geq 0$ for all $x \in K$. When K is a cone, this becomes a complementarity problem. In this talk, we describe some results connecting the variational inequality problem VI(f, K, q) and the cone complementarity problem VI$(f^\infty, K^\infty, 0)$, where f^∞ is the recession part of f and K^∞ is the recession cone of K. As an application, we discuss the solvability of nonlinear equations corresponding to weakly homogeneous maps over closed convex cones.